SELF-HEATING OF A VISCOUS FLUID UNDER CYCLIC DEFORMATION

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The liberation of heat is the most important feature of the deformation process. The thermal flow modes have been investigated well only in the case of simple shear strain, whence interesting phenomena were detected: the hydrodynamic analogs of a thermal explosion [1-3], inflammation and extinction [4, 5], ignition [6, 7], nonisothermal self-oscillation of viscoelastic fluids [8]. Interest arises in an investigation of the influence of cyclic deformation on the dissipative heating and its associated phenomena. This question, which has been studied well for solid polymers [9-12], has practically not been investigated for flow systems although it is of interest in a number of applied problems of chemical technology (vibration stamping, vibration conveying, etc.) and viscosimetry.

1. The self-heating of flowing systems under cyclic deformation is investigated theoretically in this paper in the model of a rotation vibration viscosimeter. The nonisothermal shear flow of a Newtonian fluid between two coaxial infinite cylinders, one of which rotates at a constant speed while the other performs forced tangential vibrations, is examined. The mathematical description of the process contains the motion and heat balance equations

$$\rho\frac{\partial v}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\sigma),\quad \sigma=\mu_0 e^{-\beta(T-T_0)}r\frac{\partial}{\partial r}\left(\frac{v}{r}\right); \tag{1.1}$$

$$c\rho\frac{dT}{dt}=q-\frac{2\alpha}{r_2-r_1}(T-T_0),\quad q=\frac{2}{r_2^2-r_1^2}\int_{r_1}^{r_2}\sigma r\frac{\partial}{\partial r}\left(\frac{v}{r}\right)r\,dr, \tag{1.2}$$

where v is the flow velocity, σ is the shear stress, T is the temperature, t is the time, $r_1 \le r \le r_2$, r_1, r_2 are the cylinder radii, c is the specific heat, ρ is the density, q is the dissipation function, T_0 is the temperature of the environment, α is the coefficient of heat transfer from the surface, μ_0 is the viscosity for $T=T_0$, and β is the temperature coefficient of viscosity.

The Reynolds dependence of the viscosity on the temperature $\mu=\mu_0\exp[-\beta(T-T_0)]$ taken in (1.1) is obtained by expanding the exponent of the Arrhenius dependence $\mu=A\exp(B/T)$ [13] (A and B are constants) by the Frank–Kamenetskii method [14] under the condition $(T-T_0)/T_0 \ll 1$. Equation (1.2) assumes the absence of a temperature distribution over the fluid volume; however, such a model does not lose meaning even in the presence of a temperature distribution over the volume. In this case (1.2) should be understood as an approximate temperature relative to the average over the volume.

The boundary conditions in the form

$$r=r_1\ \ v=r_1\varphi_0\omega\cos\omega t=v_1\cos\omega t,\ \ r=r_2\ \ v=v_2; \tag{1.3}$$

$$r=r_1\ \ \sigma=\sigma_1+\sigma_0\sin\omega t,\ \ r=r_2\ v=v_2. \tag{1.4}$$

are of the greatest interest.

In the first case a sinusoidal change in the angular displacement is kept in mind (v is the time derivative of the displacement, φ_0 is the angular displacement amplitude, ω is the frequency of oscillation), and a sinusoidal change in the shear stress in the second case (σ_1 is the mean value of the shear stress and σ_0 is the amplitude of the oscillations).

It is convenient to separate the characteristic times of the problem for the analysis: $t_1=c\rho(r_2-r_1)/2\alpha$ is the time of heat elimination, $t_2=c\rho/\beta q_0$ is the time of heat liberation, q_0 is the dissipation function in the stationary mode at the temperature T_0 in the absence of vibrations, $t_3=2\pi r_2/v_2$ is the time of one rotation of the outer cylinder, $t_4=2\pi/\omega$ is the period of oscillations, and $t_5=\rho(r_2-r_1)^2/\mu_0$ is the time of hydrodynamic stabilization.

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Comparing the characteristic times of the processes, we can judge their intensities and make certain a priori deductions by means of numerical estimates. For strongly viscous fluids it can be assumed that the hydrodynamic stabilization time is substantially less than all the other times. In this case a stationary equation of motion can be used whose solution has the form

$$v = (v_2 - v_1 \cos \omega t)\left(\frac{1}{1-(r_1/r_2)^2}\frac{r}{r_1} - \frac{1}{(r_2/r_1)^2-1}\frac{r_1}{r}\right),$$

$$D = r\frac{\partial}{\partial r}\left(\frac{v}{r}\right) = 2\left(\varphi_0\omega\cos\omega t - \frac{v_2}{r_2}\right)\frac{1}{r_1^{-2}-r_2^{-2}}\frac{1}{r^2}$$

(D is the strain rate) for the boundary condition (1.3), and

$$v = v_2\frac{r}{r_2} + \frac{1}{2}\frac{\sigma_1+\sigma_0\sin\omega t}{\mu_0}e^{\beta(T-T_0)}\left(\frac{r}{r_2^2} - \frac{1}{r}\right),$$

$$\sigma = (\sigma_1 + \sigma_0 \sin \omega t)\frac{1}{r^2}$$

for the boundary condition (1.4).

2. Let us consider the problem under the boundary conditions (1.3). By evaluating the dissipation function we arrive at one heat balance equation

$$\frac{d\Theta}{dt} = \frac{1}{t_2}(\gamma\cos\omega t - 1)^2 e^{-\Theta} - \frac{1}{t_1}\Theta, \tag{2.1}$$

$$\Theta = \beta(T-T_0), \quad \gamma = \frac{v_1}{v_2} = \frac{t_3}{t_4}, \quad \frac{1}{t_2} = \frac{\beta}{c\rho}q_0 = \frac{\beta}{c\rho}\frac{2\mu_0 v_2^2 r_1^2}{(r_2^2-r_1^2)\,r_2^2}.$$

The essential flow singularities can be clarified from an analysis of the different limit relationships between the characteristic times. From the viewpoint of the influence of vibration on self-heating, the two limit cases which are analyzed below are of special interest.

1. Case of "Fast Oscillations" $t_1 \gg t_4$. Let us take t_4 ($\tau = t/t_4$) as the time scale. Then (2.1) is written as

$$\frac{d\Theta}{d\tau} = \frac{t_1}{t_2}\left(\gamma\cos 2\pi\frac{t_1}{t_4}\tau - 1\right)^2 e^{-\Theta} - \Theta. \tag{2.2}$$

Because $t_1/t_4 \gg 1$, the right side of the equation is a rapidly oscillating function of τ. According to the averaging method [15], the approximate solution (to t_4/t_1 accuracy) of (2.2) can be obtained from the averaged equation

$$\frac{d\Theta}{d\tau} = \frac{t_1}{t_2}\left(\frac{\gamma^2}{2} + 1\right)e^{-\Theta} - \Theta. \tag{2.3}$$

The physical meaning of the averaging used is that if a large number of cycles is contained in unit time, then the temperature change within each cycle, and therefore, the viscosity change also, will be insignificant and can be considered constant. The solution of (2.3) has a simple structure (Fig. 1, curve 1). As $\tau \to \infty$ it approaches the stationary value monotonically, which can be found from the relationship

$$\Theta e^{\Theta} = \frac{t_1}{t_2}\left(\frac{\gamma^2}{2} + 1\right). \tag{2.4}$$

This formula displays the influence of the parameter γ, characterizing the oscillation intensity, on the dissipative heating. In the absence of oscillations (for $\gamma = 0$), this formula agrees with the known formula from [2], obtained for the computation of the self-heating of a fluid in a simple shear flow.

The solution of (2.2) performs particular oscillations in the background of the solution of the averaged equation (2.3); the amplitude of these oscillations is hence small (on the order of t_4/t_1) independently of the parameter γ.

2. Case of "Strong Heat Elimination" $t_1 \ll t_4$. Let us take t_4 ($\tau = t_1/t_4$) as the time scale and let us rewrite (2.1) in the form

$$\frac{t_1}{t_4}\frac{d\Theta}{d\tau} = \frac{t_1}{t_2}(\gamma\cos 2\pi\tau - 1)^2 e^{-\Theta} - \Theta. \tag{2.5}$$

Because $t_1/t_4 \ll 1$, the self-heating $\Theta(\tau)$ varies quasistationarily. This assertion results from the Tikhonov theorem [16], according to which (2.5) is close (to t_1/t_4 accuracy) to the solution of the degenerate equation

$$\Theta e^{\Theta} = (t_1/t_2)(\gamma\cos 2\pi\tau - 1)^2,$$

governing the function $\Theta(\tau)$ which is periodic in τ with period 1 outside a certain boundary layer abutting on $\tau = 0$. The behavior of the solution is different for $\gamma \geq 1$ ($v_1 \geq v_2$) and $\gamma < 1$ ($v_1 < v_2$) (see Fig. 1, curves 2 and 3). For

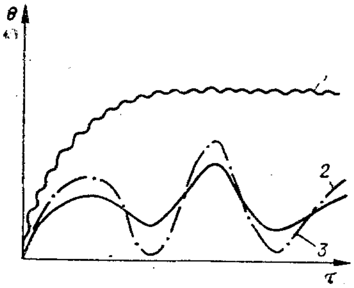

Fig. 1

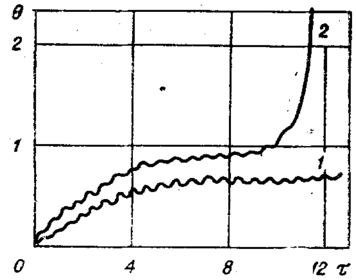

Fig. 2

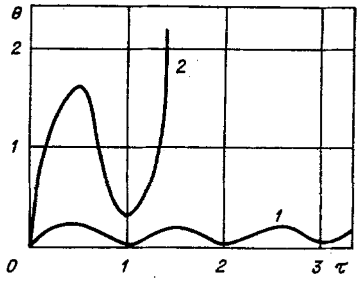

Fig. 3

$\gamma \geq 1$ the heat source is disconnected at individual times when the cylinder velocities are equal, and the heating drops almost to zero (to t_1/t_4 accuracy) because of the strong heat elimination.

The heat liberation maximums correspond to the extreme positions of the oscillating cylinder, where a greater value of the heat liberation is reached for oppositely directed cylinder velocities (greater humps on curves 2 and 3 in Fig. 1). The small humps on the same curves correspond to identically directed cylinder velocities. For $\gamma < 1$ the heat source is not disconnected and the minimum heating becomes essential:

$$\min \Theta = \Theta_1, \quad \Theta_1 e^{\Theta_1} = \frac{t_1}{t_2}(1-\gamma)^2.$$

The mean value of the function $\Theta(\tau)$ over the period can be found approximately for not very large amplitudes by solving (2.4). Here the approximate average of the nonlinear function $F(\Theta) = \Theta \exp \Theta$ by means of the formula [17] $\langle F(\Theta)\rangle = F(\langle\Theta\rangle)$ is kept in mind.

The domain of application of the results obtained for the "fast oscillations" and "strong heat elimination" cases also includes the other limit cases (for instance, $t_1 \ll t_2$ and $t_3 \ll t_4$) since the asymptotic solutions obtained are valid uniformly in t_1/t_2, t_3/t_4 in any finite interval of the form [0, M]. Let us note that the limit relationship $t_2 \ll t_1$ always corresponds to the adiabatic case described by (2.1) in the absence of heat elimination. The solution of the adiabatic equation under the initial condition $\Theta(0) = \Theta_0$ has the form

$$e^{\Theta} = e^{\Theta_0} + \frac{1}{t_2}\left(t + \frac{\gamma^2 t}{2} + \frac{\gamma^2}{4\omega}\sin 2\omega t \frac{2\gamma}{\omega}\sin \omega t\right).$$

It hence follows that the adiabatic temperature, which fluctuates, grows logarithmically. An increase in the frequency diminishes the span of the oscillations and increases the rate of temperature rise.

3. Let us consider the problem under the boundary conditions (1.4). Evaluating the dissipation function in this case, we arrive at the following form of the heat balance equation:

$$\frac{d\Theta}{dt} = \frac{1}{t_2}(\gamma_1 \sin \omega t + 1)^2 e^{\Theta} - \frac{1}{t_2}\Theta, \tag{3.1}$$

$$\Theta = \beta(T - T_0), \quad \gamma_1 = \frac{\sigma_0}{\sigma_1}, \quad \frac{1}{t_2} = \frac{\beta}{c\rho} q_0 = \frac{\beta}{c\rho}\frac{\sigma_0^2}{r_1^2 r_2^2 \mu_0}.$$

In this boundary-conditions case the velocity v_2 does not influence the self-heating, and hence there is no time t_3 here.

Without taking account of the periodicity of the source, (3.1) is well known in the theory of a thermal explosion [14]. Depending on the parameters, the presence of two qualitatively distinct types of solutions, explosive and nonexplosive, was shown for it. The presence of a periodic time factor in the dissipation function introduces an essential singularity. By analogy with the previous case, let us consider the fundamental limit relationships of the characteristic times.

1. For $t_1 \gg t_4$ equation (3.1) becomes for the dimensionless time $\tau = t/t_1$

$$\frac{d\Theta}{d\tau} = \frac{t_1}{t_2}\left(\gamma_1 \sin 2\pi \frac{t_1}{t_4}\tau + 1\right)^2 e^{\Theta} - \Theta. \tag{3.2}$$

Let us again use the method of averaging [15] to solve this equation with a rapidly oscillating right side. To t_4/t_1 accuracy, the solution of (3.2) is close to the solution of the averaged equation

$$\frac{d\Theta}{d\tau} = \varkappa e^{\Theta} - \Theta, \quad \varkappa = \frac{t_1}{t_2}\left(\frac{\gamma_1^2}{2} + 1\right).$$

For $\varkappa < \varkappa_* = 1/e$ the nonexplosive, low-temperature flow mode holds, while for $\varkappa \gg \varkappa_*$ a progressive growth of self-heating, a hydrodynamic thermal explosion, occurs.

It is interesting to note that the critical condition

$$\varkappa_* = \frac{t_1}{t_2}\left(\frac{\gamma_1^2}{2} + 1\right) = \frac{1}{e} \tag{3.3}$$

contains only amplitude values of the shear stress and does not contain the frequency of oscillation. At the same time, the critical condition occurring under the cyclic deformation of solid polymers with a given stress amplitude [9-12] in this same limit case does contain the frequency. This distinction is related to the fact that the properties of a solid (pliability of loss or modulus of loss) depend on the external deformation parameters (the frequency) and there is no such dependence for a Newtonian fluid. In the absence of vibrations ($\gamma_1 = 0$) the critical condition (3.3) goes over into the equality $\varkappa_* = t_1/t_2 = 1/e$, which is well known from the theory of the thermal explosion. The regularities mentioned are illustrated in Fig. 2, in which curves of the dependence $\Theta(\tau)$ are displaced which have been obtained from numerical computations of (3.2) on an electronic computer (curve 1 corresponds to parameters of the low-temperature domain $t_1/t_4 = 10.0$, $t_1/t_2 = 0.214$, $\gamma_1 = 1.0$, $\varkappa = 0.32062 < \varkappa_*$; 2 to the parameters of the explosion domain $t_1/t_4 = 10.0$, $t_1/t_2 = 0.260$, $\gamma_1 = 1.0$, $\varkappa = 0.39141 > \varkappa_*$).

2. For $t_1 \ll t_4$, by taking t_4 as the time scale we arrive at the following equation:

$$\frac{t_1}{t_4}\frac{d\Theta}{d\tau} = \frac{t_1}{t_2}(\gamma_1 \sin 2\pi\tau + 1)^2 e^{\Theta} - \Theta. \tag{3.4}$$

Let us consider $f(\tau) = (t_1/t_2)(\gamma_1 \sin 2\pi\tau + 1)^2$, for which

$$f_1 = \frac{t_1}{t_2}(1 + \gamma_1)^2, \quad f_0 = \begin{cases} (t_1/t_2)(1 - \gamma_1)^2, & \gamma_1 < 1, \\ 0, & \gamma_1 \geqslant 1 \end{cases}$$

denote the maximum and minimum values, respectively. According to the Tikhonov theory [16], for $f_1 < 1/e$ the periodic heating $\Theta(\tau)$ is set up to t_1/t_4 accuracy, which has been defined as the smaller root of the equation

$$\Theta e^{-\Theta} = \frac{t_1}{t_2}(\gamma_1 \sin 2\pi\tau + 1)^2. \tag{3.5}$$

If $f_0 > 1/e$ (this is possible for $\gamma_1 < 1$), then a quasistationary solution is not possible. Physically this means a hydrodynamic thermal explosion. According to (3.5) a quasistationary solution exists in the intermediate regimes for $f_1 > 1/e > f_0$ but only on specific sections of the variation in τ (for $f(\tau) < 1/e$). There is no such solution for $f(\tau) > 1/e$). A numerical analysis of (3.4) showed that an explosive mode is also characteristic for this domain. Results of a numerical computation are presented in Fig. 3. Curve 1 corresponds to the nonexplosive domain of parameters ($t_1/t_4 = 0.10$, $t_1/t_2 = 0.100$, $\gamma_1 = 0.5$, i.e., $f_1 < 1/e$), 2 to the transition domain ($t_1/t_4 = 0.25$, $t_1/t_2 = 0.421$, $\gamma_1 = 0.5$, i.e., $f_1 > 1/e > f_0 = 0.2625$). In the limit as $(t_1/t_4) \to 0$, the critical condition of the explosion is expressed by the formula

$$f_1 = \frac{t_1}{t_2}(1 + \gamma_1)^2 = \frac{1}{e}.$$

Let us note that this critical condition is associated with disruption of the maximum temperature, while it is determined by the disruption of the mean temperature in the limit case of "fast oscillations" (see (3.3)).

The period of induction of the explosion τ_i can be defined as the time of the first disruption of the quasistationary solution in the domain

$$t_1/t_2 < 1/e, \; f_1 > 1/e \tag{3.6}$$

i.e., as the least solution of the equation $f(\tau) = 1/e$. Taking account of $\tau_i = t_i/t_4$ this yields

$$t_i = \frac{t_4}{2\pi} \arcsin\left[\left(\sqrt{\frac{t_2}{t_1 e}} - 1\right)/\gamma_1\right]. \tag{3.7}$$

It follows from the formula that the period of induction for the case $(t_1/t_4) \to 0$ is much less than the period of the oscillations. Hence, disruption of the temperature occurs within the first cycle, and condition (3.6) defines the explosion domain. For $t_1/t_2 > 1/e$ formula (3.7) does not define the period of induction. Within the framework of the assumption made about the smallness of t_1/t_4, this means that $\tau_i = O(t_1/t_4)$.

Let us note that, as in Sec. 2, the asymptotic solutions obtained are also valid for other limit cases. In the adiabatic case $(t_2 \ll t_1)$, we have for $\Theta(0) = \Theta_0$

$$e^{-\Theta} = e^{-\Theta_0} - \frac{1}{t_2}\left[\left(1 + \frac{\gamma_1^2}{2}\right)t + \frac{2\gamma_1}{\omega}(1 - \cos\omega t) - \frac{\gamma_1^2}{4\omega}\sin 2\omega t\right].$$

The present examination is directed towards studying the qualitative regularities of the self-heating of flowing systems under cyclic deformation. To this end, the simplest model of a Newtonian fluid was selected, whose properties are independent of the vibration parameters. From the viewpoint of determining the rheological properties of a fluid from dynamic tests, the investigation of self-heating of non-Newtonian fluids, whose properties, besides the temperature, can also depend on the oscillation frequency, is of interest.

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